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by

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ABSTRACT

Production decisions in neoclassical, certainty models of capital investment by firms have been the driving force behind current theoretical specifications of investment behavior. Often a specific form of the production technology is assumed. Recently the concept of costs of adjustment has been included in these models. When properly specified, this cost function yields a unique, optimum firm size (i.e., a determinate level of output and investment) and theoretically justifies the use of distributed lags in econometric analysis.

This paper develops a more general model of production and investment decisions made in a world of uncertainty wherein firm values are determined in an explicit capital market. With less restrictive assumptions as to technology, and both with and without costs of adjustment, we derive theoretical results which are empirically identical to the earlier research described above. Thus we also bring into question which model the previous empirical analyses actually validated. Further, we show rigorously that the investment and output levels of a firm in our uncertain world, with non-competitive aspects of the capital market, will be strictly less than those in a certainty context. With a competitive capital market under uncertainty, however, investment and output levels are shown to equal those of a certain world.

Jorgenson [19] presents an exhaustive treatment of the neoclassical model. Therefore, the results of our analysis are compared to his.

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1. INTRODUCTION

An important segment of the literature on neoclassical investment models of the firm is based on the explicit existence of causal interdependence among projects in the form of "costs of adjustment" to capital.[1] The theoretical constructs and empirical analyses of this literature claim to explain international and domestic U.S. corporate investment. In particular the assumption of a strictly convex costs of adjustment function is claimed[2] to be an important, theoretically valid reason for the use of distributed lags in econometric analysis of investment. Further, many of these econometric results cannot reject the hypothesis that investment decisions are made in the context of a Constant Elasticity of Substitution (CES) production function with elasticity of substitution equal to unity and constant returns to scale (i.e., a Cobb-Douglas production function) in a world of certainty.

Jorgenson [19] is the main proponent of this econometric research and his analysis claims to explain the discrepancies among many of the major empirical studies of investment behavior in neoclassical models. He shows that proper treatment of costs of adjustment in all these other models would modify their results such that the Cobb-Douglas form holds. However, Jorgenson's work and that of the others he criticizes are partial equilibrium analyses in that they ignore financial market considerations, concentrating instead solely on production decisions of the firm. The entirety of their work is done in a certainty context, whereas the inclusion of financial market valuation of projects explicitly incorporates uncertainty.

Our objective in this paper is to bring production and finance together in the following manner. We compare Jorgenson's results to those obtained in a world of uncertainty where firms' values are explicitly determined in a capital market. We use Occam's razor to explore the validity of the costs of adjustment concept and the other assumptions of Jorgenson's model. At first we follow Jorgenson's basic assumptions in our model so as to exactly compare results. Later, we drop these restrictive assumptions and use a more general model to derive similar results. We find that our model of a value-maximizing firm making decisions in a world of uncertainty as specified by the Capital Asset Pricing Model (CAPM)[3] yields identical results to those obtained for output/capital and output/labor ratios in Jorgenson's model. These CAPM results are shown to hold both with and without costs of adjustment. Thus we bring into question which model Jorgenson's analysis actually validates.

2. COSTS OF ADJUSTMENT AND CERTAINTY

In a security market equilibrium with causal independence, value additivity[4] implies the market value of a firm is equal to the sum of its separate projects valued independently. In particular the firm can make its new project investment decisions without regard to the other projects it holds or is considering. Myers [31] and, later, Schall [35] (in an arbitrage proof) and Rubinstein [34] show that causal independence exists in their proofs that value additivity holds in security market equilibrium. However, the costs of adjustment models claim that any project investment decision _necessarily_ imposes additional costs on the firm as a function of other projects. Jorgenson assumes these costs are internal to the firm and may take various forms. Their common attribute, though, is that they reflect a loss of cash flow from existing projects' assets. For example, shutting down the existing production line while installing new machinery or incurring overtime costs to keep output level. Costs of adjustment imply causal dependence among projects in intertemporal investment decisions. Thus all firms explicitly consider these interaction costs to existing and potential cash flows when deciding on investment in a new (or expanded old) project. Jorgenson claims these costs are of a meaningful magnitude and exist in the general case in the international economy, and that his econometric analysis supports this. The implications are many. For example, differing costs of adjustment functions across firms can give rise to a theory of conglomerate merger in a sequential economy.

Lucas [25], Uzawa [41] and Jorgenson [19] postulate a function, $F(K_o,L)$, which represents the beginning of period production technology available to the firm with factor inputs K_o and L. Labor, L, is assumed variable whereas capital, K_o, is the amount owned by the firm at the end of the last period and carried over to the beginning of the current period. This capital, K_o, is "fixed" --i.e., it is a "clay" technology. When a firm undertakes new investment in this period, these researchers can write $K = K_o + I$ where I is the gross amount of new investment, if any, taken on in the period. Then $F \equiv F(K,L)$ where L now includes that additional labor necessary to work with I. However, they also assume there exist costs of adjustment associated with taking on I. These costs are a function of the interrelationship between the existing capital stock and the new investment. They assume these costs of adjustment are additively separable from production, F, and thus represent them as $G \equiv G(K,I)$. Jorgenson further assumes these functions, F and G, are homogeneous of degree one in their arguments and writes the firm's physical output less the costs of adjustment as:

$$F(K,L) - G(K,I) \tag{1}$$

where:

$$G(K,0) = 0$$

$$F_K > 0, \quad F_L > 0, \quad G_K < 0, \quad G_I > 0$$

$$F_{KK} < 0, \quad F_{LL} < 0, \quad G_{KK} > 0, \quad G_{II} > 0$$

The assumption that G is strictly convex in I is an important one. As shown rigorously by Rothschild [33], a profit-maximizing, competitive[5] firm facing a strictly convex cost of adjustment function will not take on all the project's requisite investment in one period. Rather, the firm's optimal investment program will be to distribute the investment over all subsequent periods. This gives the justification for distributed lags in econometric analysis of investment behavior. Additionally, this assumption gives rise to a determinate firm size as of a given period, both in terms of investment and output (for any positive price vector). However, if the G function is linear or concave, all investment occurs in the immediate period and firm size is again indeterminate in the neoclassical competitive case.

We adapt Jorgenson's model as follows. Assume a world of certainty and perfect competition in product and factor markets where firm j owns fixed[6] capital goods, in amount K_o, at the beginning of the period (t=0). At this time it makes its production decision by choosing how much of K_o to activate, how much L to employ at known factor cost w, and how much new investment, I, to make.[7] During the period, K_o and I suffer a percentage cost, δ (where $0 \leq \delta \leq 1$), of economic depreciation and all the firm's remaining assets, $(1-\delta)(K)$, can be liquidated at the end of the period (t=1) at a known equilibrium unit price v.[8] The I is expended at the beginning of the period, but the cash inflow from sale of net output and assets, and the factor payment to L occur at the end of the period. Thus the net present value, V_j, of the firm can be written as:

$$V_j = \frac{PF(K,L) - PG(K,I) - wL + v(1-\delta)K}{R} - I \qquad (2)$$

where R is unity plus the riskless rate, r. Since we have incorporated the technological constraint, the value-maximizing (to beginning of period equityholders) firm's investment criterion is the unconstrained maximization of (2) with respect to the decision variables K_o, I and L. Note that any amount of existing capital can be employed up to the amount K_o. This represents an implicit constraint and the concept of excess capacity is perfectly well-defined. Since the marginal cost of employing K_o is zero, we would expect the constraint to be binding. The certain output price, P, and other parameters, r, w, v and δ are taken by the firm as exogenously given.

The introduction of costs of adjustment in (2) results in an interior optimum when (2) is maximized. Thus Jorgenson derives a determinate level of investment and a determinate scale of output in a given period for any positive price vector, even with a constant returns to scale (CRTS) production technology. The optimal levels of firm investment and output depend on its existing, fixed capital stock, K_o. It is important to understand this. Effectively, the existence of these (internal) increasing costs of adjustment causes the firm to perceive a rising cost of capital even though the (external) capital market is competitive.

The necessary conditions for (2) to be maximized[9] are:

$$\frac{\partial V}{\partial K} = 0 = \frac{PF_K - PG_K - PG_I + v(1-\delta)}{R} - 1 \tag{3}$$

$$\frac{\partial V}{\partial L} = 0 = \frac{PF_L - w}{R} \tag{4}$$

Equation (4) yields the standard competitive result in a certain world that the marginal product of labor is equal to its real wage, i.e.,

$$F_L = \frac{w}{P} \tag{5}$$

We will have more to say about the role of labor later.

Previous empirical work,[10] without the specification of costs of adjustment, tested the standard competitive result that the marginal product of capital was equal to its real user cost, i.e., $F_K = \frac{R - v(1-\delta)}{P}$. Using a CES production function, this previous work yielded widely differing results as to the elasticity of substitution[11] between factors. The resulting values of this key parameter varied from near zero to somewhat greater than unity. Jorgenson claims that the introduction of costs of adjustment to capital reconciles all these previous discrepancies, results in a value of unity for the elasticity of substitution, and that his resultant Cobb-Douglas production function explains international, as well as domestic U.S., firm investment.

Following Jorgenson we may write (3) as follows:

$$F_K = \frac{R - v(1-\delta)}{P} + G_K + G_I$$

$$\Longrightarrow F_K = \left[\frac{R - v(1-\delta)}{P}\right]\left[1 + \frac{PG_I}{R - v(1-\delta)} + \frac{PG_K}{R - v(1-\delta)}\right] \tag{6}$$

Equation (6) is identical to that obtained by Jorgenson ([19] page 234) for the marginal product of capital in a CES production function assuming constant returns to scale. Obviously without costs of adjustment, represented by $G_I + G_K$, equation (6) becomes the standard competitive result:

$$F_K = \frac{R - v(1-\delta)}{P} \tag{7}$$

The firm in Jorgenson's model chooses investment in the new project so as to satisfy (6). Further, we may note the following characteristics of F_K:

$$\frac{\partial F_K}{\partial r} > 0,\ \frac{\partial F_K}{\partial P} < 0,\ \frac{\partial F_K}{\partial v} < 0,\ \frac{\partial F_K}{\partial \delta} > 0$$

which imply: (8)

$$\frac{\partial I}{\partial r} < 0,\ \frac{\partial I}{\partial P} > 0,\ \frac{\partial I}{\partial v} > 0,\ \frac{\partial I}{\partial \delta} < 0$$

as we would expect.

In a fashion similar to Jorgenson ([19] page 234) we may now transform (6) into an empirically testable equation where there exist costs of adjustment. Recall Jorgenson's assumption of constant returns to scale. By Euler's theorem we may then write:

$$F = KF_K + LF_L \Longrightarrow F_K = \frac{F}{K} - \frac{LF_L}{K} = \frac{F}{K}\left[1 - \frac{LF_L}{F}\right] \tag{9}$$

Now substituting from (4), we can write (9) as:

$$F_K = \frac{F}{K}\left[1 - \frac{wL}{PF}\right] \tag{10}$$

Substituting from (10) into (6) yields:

$$\frac{F}{K}\left[1 - \frac{wL}{PF}\right] = \left[\frac{R-v(1-\delta)}{P}\right]\left[1 + \frac{PG_I}{R-v(1-\delta)} + \frac{PG_K}{R-v(1-\delta)}\right] \tag{11}$$

Taking natural logs of both sides in (11) and rearranging terms gives:

$$\ln\left(\frac{F}{K}\right) = -\ln\left[1 - \frac{wL}{PF}\right] + \ln\left[\frac{R-v(1-\delta)}{P}\right] + \ln\left[1 + \frac{PG_I}{R-v(1-\delta)} + \frac{PG_K}{R-v(1-\delta)}\right] \tag{12}$$

Jorgenson derives his empirically specified equation for the log of the output/capital ratio as follows. He writes the last term in brackets of the right hand side (RHS) of (12) as A, thus:

$$A \equiv 1 + \frac{PG_I}{R-v(1-\delta)} + \frac{PG_K}{R-v(1-\delta)} \tag{13}$$

The variable A represents the effect of costs of adjustment. Note that A has the following characteristics[12] when $I > 0$ for a given firm size:

$$A > 1, \quad \frac{\partial A}{\partial K} > 0, \quad \frac{\partial A}{\partial I} > 0$$

Note that $1 - \frac{wL}{PF}$, the first term on the RHS of (12), is actually the relative share of capital in this model as shown in our Appendix. That is, Jorgenson assumes[13] a production function with constant returns to scale, i.e., homogeneous of degree one. By Euler's theorem we know that a linear homogeneous production function is a sufficient condition for the exact exhaustion of the product -- that total real payments to factors (here L and K) sum to total quantity of real product, F. Thus:

$$F = \frac{w}{P} L + \frac{R-v(1-\delta)}{P} K \tag{14}$$

in our model. Letting $Z \equiv R-v(1-\delta)$, the cost of capital services, we may rewrite (14) as follows, (15), to express the relative share of capital:

$$\frac{ZK}{PF} = 1 - \frac{wL}{PF} \tag{15}$$

It is well known that relative factor shares in this type of model are constant if the elasticity of substitution is unity, and vice versa. Thus we must be very careful in an econometric analysis of this model since assuming $-\ln\, [1 - \frac{wL}{PF}]$ as the constant term in an OLSQ regression is implicitly assuming the elasticity of substitution to be unity; but this elasticity is a parameter we wish to estimate.

Jorgenson imposes the assumption of CES, which, applied to equation (12), eliminates the relative share of capital as the first term on the RHS. He can then derive a constant term[14] of:

$$\eta[\rho \ln \gamma - \ln \delta] \equiv C \tag{16}$$

where, as stated parameters in his assumed CES function, the coefficient $\eta (\equiv 1/(1+\rho))$ is the elasticity of substitution, ρ is the substitution parameter, γ is the efficiency parameter, and δ is the distribution parameter. Jorgenson correctly treats this term, (16), as his regression constant because under his assumption of CES, (16) is not the relative share of capital unless $\rho = 0$ is also assumed.

Assuming a CES production technology, we may now specify an OLSQ estimation of (12), identical to Jorgenson's, as the following:

$$\ln\left(\frac{F}{K}\right) = C + \eta[\ln\left(\frac{Z}{P}\right) + \ln A] + \varepsilon_K \tag{17}$$

where C is the constant term, η is the regression coefficient representing the elasticity of substitution, ε_K is the error term, and where Jorgenson assumes $\ln\left(\frac{Z}{P}\right)$, ln A, and ε_K are each distributed independently. Equation (17) is identical to Jorgenson's analysis of the Arrow et. al. [1], Dhrymes-Zarembka [10], Fuchs [15], Griliches [17], and Bell [2] models wherein costs of adjustment for capital are now explicitly included, thus correcting the claimed specification error in those previous empirical tests. Comparing analyses of cross-sectional data, both international and U.S. (by state), Jorgenson concludes that η is not significantly different from unity, thus implying a Cobb-Douglas function.[15] He then restates (17) as:

$$\ln\left(\frac{Z}{P}\right) = C + \frac{1}{\eta}\ln\left(\frac{F}{K}\right) - \left[\ln A + \frac{1}{\eta}\varepsilon_K\right] \tag{18}$$

Equation (18) is identical to the original Dhrymes [9] equation but now including costs of adjustment. Again Jorgenson claims η is not significantly different from unity when the negative correlation of the error term with the independent variable in (18) is accounted for. Note that in (18) he assumes the total error term is: $-\left[\ln A + \frac{1}{\eta}\varepsilon_K\right]$, that is it includes the mispecification of costs of adjustment.

3. UNCERTAINTY

Let us now consider a world in which the value-maximizing firm makes investment decisions under uncertainty. Assuming the CAPM holds intertemporally,[16] we may write the market equilibrium value[17] of that firm, V_j, as:

$$V_j = \frac{1}{R}\,[\bar{X}_j - \lambda\sigma_{jM}] \tag{19}$$

where $\bar{X}_j$ is the expected end of period cash flow, σ_{jM} is the covariance of firm j's cash flows with those of the market (all firms), and λ is the (constant) market-determined price of risk. For simplicity we assume the only source of uncertainty in this model comes from consumers' tastes through output demand. Thus a random output price, $\tilde{P}$, represents this uncertainty to the competitive price-taking firm. This price is revealed at the end of the period at which time all the firm's output is sold and hired-factor payments made at the known wage. Production decisions, including investment, are made at the beginning of the period.

Now, as in Jorgenson's model, suppose this firm also has an investment opportunity to decide upon at the beginning of the period, however there are no costs of adjustment to capital. We assume startup costs, etc. to be included in the initial investment amount. We may write the firm's end of period cash flow, $\tilde{X}_j$, after taking the investment decision where, as with Jorgenson, $K = K_o + I$ as:

$$\tilde{X}_j = \tilde{P}F(K,L) - wL + v(1-\delta)K \tag{20}$$

Let $E(\tilde{P}) \equiv \overline{P}$, then:

$$E(\tilde{X}) \equiv \overline{X}_j = \overline{P}F(K,L) - wL + v(1-\delta)K \tag{21}$$

where E is the expected value operator. Also it can be shown that:

$$\begin{aligned}\sigma_{jM} &= \operatorname{cov}[\tilde{P}F(K,L) - wL + v(1-\delta)K,\ \tilde{X}_M + \tilde{P}F(K,L) - wL + v(1-\delta)K] \\ &= F\sigma_{PM} + F^2\sigma_P^2\end{aligned} \tag{22}$$

where $\tilde{X}_M$ is the cash flow of all the other firms in the market.[18] Substituting (21) and (22) into (19) yields the value of the firm at the beginning of the period:

$$V_j = \frac{1}{R}\,[\overline{P}F(K,L) - wL + v(1-\delta)K - \lambda(F\sigma_{PM} + F^2\sigma_P^2)] \tag{23}$$

Our objective is to maximize firm value net of the requisite investment, i.e., $V_j - I$. Thus subtracting I from the right hand side of (23) gives:

$$V_j = \frac{1}{R}\,[\overline{P}F(K,L) - wL + v(1-\delta)K - \lambda(F\sigma_{PM} + F^2\sigma_P^2)] - I \tag{24}$$

The necessary conditions for a maximum of (24) are:

$$\frac{\partial V_j}{\partial L} = 0 = \frac{\overline{P}F_L - w - \lambda(F_L\sigma_{PM} + 2FF_L\sigma_P^2)}{R} \tag{25}$$

$$\frac{\partial V_j}{\partial K} = 0 = \frac{\overline{P}F_K + v(1-\delta) - \lambda(F_K\sigma_{PM} + 2FF_K\sigma_P^2)}{R} - 1 \tag{26}$$

We may solve (26) for F_K and write:

$$F_K = \frac{R - v(1-\delta)}{\overline{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)} \tag{27}$$

Equation (27) is an important result. Note that it is equivalent to the standard competitive result under certainty, (7), except for expected price and a risk premium in the denominator. This is as we would expect. For the comparative analysis that follows we assume output price in the certainty model, P, equals $\overline{P}$, expected price in our uncertain world. Thus F_K in (27) is greater than F_K in (7) which implies that both investment and output are strictly less in an uncertain world than in a certain one due to the necessity of a premium for risk.[19] Moreover, (27) is qualitatively identical to Jorgenson's result, (6), but of course (27) makes no mention of costs of adjustment. In particular (27) exhibits the same characteristics (as shown in (8)) as Jorgenson's model, (6). Thus without costs of adjustment (much less their additive separability), without assuming a specific form, e.g., CES, for the production function, and without the additional restriction of certainty, we are able to exactly replicate Jorgenson's results in our world where assetholders value their financial claims according to the CAPM.

There is an important caveat. As discussed previously, the introduction of costs of adjustment allowed Jorgenson to derive an interior maximum for (2) from conditions (3) and (4), and thus gives a theoretical justification for lagged investment behavior. Likewise, the introduction of uncertainty allows us to also reach an interior maximum and thus a determinate firm size -- i.e., determinate levels of (lagged) investment and output in any given period -- from conditions (25) and (26). Our value equation (24) is not homogeneous even though we allow constant returns to scale technology (i.e., if F is homogeneous of the first degree). This result is not due to externalities associated with uncertainty as Fama [14] surmised. The reason for this nonhomogeneity is as follows.[20] In deriving (24) from (19) to account for the new investment we have allowed the firm's perception of σ_{jM} to change (though λ and r remain constant) to, say, σ'_{jM}. This change is evident in the $\frac{\partial\sigma_{jM}}{\partial L}$ and $\frac{\partial\sigma_{jM}}{\partial K}$ terms in (25) and (26). Thus we are implicitly assuming that the firm perceives its investment decision to affect the aggregate amount of investment in the technology by all firms. That is, in the CAPM the project's equilibrium cost of capital is a monotonic function of σ_{jM}. But as in any equilibrium analysis of demand and supply, it is aggregate (across all firms) project investment which determines the price of capital services. Thus only a non-price-taker (for capital) firm can affect the project's market cost of capital and it does so by affecting aggregate project investment and thus σ_{jM}. This, of course, implies the existence of entry barriers. However, if the technology is freely available to all firms then we must assume the firm makes its investment decision taking all other firms' investment in this project as given, even with free entry. If our firm has monopolistic power over this technology, then our result, σ'_{jM}, follows straightaway. Thus the result centers on noncompetitive aspects of the capital market. This compares to the increasing cost of capital perceived by the firm facing internal costs of adjustment in Jorgenson's model even though the (external) capital market is competitive. The firm affects this cost of capital by varying its rate of project investment of which costs of adjustment are a monotonic function.

We may say more. From (25) we can rewrite (10) for our uncertain world as:

$$F_K = \frac{F}{K}\left[1 - \frac{wL}{F[\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)]}\right] \tag{28}$$

Substituting (28) into (27), multiplying the right hand side of (27) by $\frac{\bar{P}}{\bar{P}}$, and substituting for Z gives:

$$\frac{F}{K}\left[1 - \frac{wL}{F[\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)]}\right] = \frac{Z}{\bar{P}}\left[\frac{\bar{P}}{\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)}\right] \tag{29}$$

Taking the natural log of both sides of (29) and rearranging terms, we may write:

$$\ln\left(\frac{F}{K}\right) = -\ln\left[1 - \frac{wL}{F[\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)]}\right] + \ln\left(\frac{Z}{\bar{P}}\right) + \ln\left[\frac{\bar{P}}{\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)}\right] \tag{30}$$

Compare equation (30) to the certainty-world specification, equation (12). They are qualitatively very similar. The first term on the RHS of (30) is the expected relative share of capital. In particular, we may combine the first and last bracketed terms on the RHS of (30) and write them as A', thus:

$$A' \equiv \frac{\bar{P}F}{\bar{P}F - F\lambda(\sigma_{PM} + 2F\sigma_P^2) - wL} = \frac{\bar{P}F}{\bar{P}F - \lambda(\sigma_{jM} + F^2\sigma_P^2) - wL}$$

Our A' corresponds to Jorgenson's variable A in (17) representing the presumed additional costs of adjusting the capital base; however, A' derives from the firm's adjusting its capital/output ratio for impacts of uncertainty in the factor/output price ratio. Note that A' has similar qualitative characteristics as A (see (13)) when I > 0, that is:

$$A' > 1\ ,\ \frac{\partial A'}{\partial K} > 0,\ \frac{\partial A'}{\partial I} > 0$$

Then with only the same independence assumptions as Jorgenson, the OLSQ estimation of (30) would be indistinguishable from Jorgenson's (17) since (30) becomes:

$$\ln(\frac{F}{K}) = C' + \eta' \, [\ln(\frac{Z}{P}) + \ln A'] + \epsilon'_K \qquad (31)$$

where C' is a regression constant and η' is our regression coefficient.[21] Note that we have not needed to assume CES to derive (31) and our coefficient, η', is not to be interpreted as the elasticity of factor substitution. Jorgenson's specification of η in equation (17) as the elasticity of substitution depends critically on his additional maintained hypothesis of CES. Yet (31) gives the same estimation results as (17) and there is no independent test between the two specifications of CES. If we were to assume CES in our uncertainty model (31), then we have $C' = C$ and $\eta = \eta'$, thus the regression coefficient is correctly interpreted as the elasticity of substitution; however, it is dependent on the uncertainty aspects rather than costs of adjustment because A and A' are entirely different, though not mutually exclusive, concepts.

The empirical conclusions of Jorgenson, including his reconciliation[22] of all the other researchers' tests, hold equally well for our uncertainty model (31) for the output/capital ratio. This is the "identification problem" and the key question is obvious -- which model did he actually validate? Our specification from the CAPM, (31), is observationally equivalent to Jorgenson's model -- the same empirical tests would support either. In fact, with A' as a RHS variable, we would expect differing empirical estimates of η' in both cross-section and time series analyses, and for aggregate vs. micro models. For example, σ_{jM} is known to vary across time sub-periods and

We may say more. From (25) we can rewrite (10) for our uncertain world as:

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Substituting (28) into (27), multiplying the right hand side of (27) by $\frac{\bar{P}}{\bar{P}}$, and substituting for Z gives:

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Taking the natural log of both sides of (29) and rearranging terms, we may write:

$$\ln\left(\frac{F}{K}\right) = -\ln\left[1 - \frac{wL}{F[\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)]}\right] + \ln\left(\frac{Z}{\bar{P}}\right) +$$

$$+ \ln\left[\frac{\bar{P}}{\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)}\right] \qquad (30)$$

Compare equation (30) to the certainty-world specification, equation (12). They are qualitatively very similar. The first term on the RHS of (30) is the expected relative share of capital. In particular, we may combine the first and last bracketed terms on the RHS of (30) and write them as A', thus:

$$A' \equiv \frac{\bar{P}F}{\bar{P}F - F\lambda(\sigma_{PM} + 2F\sigma_P^2) - wL} = \frac{\bar{P}F}{\bar{P}F - \lambda(\sigma_{jM} + F^2\sigma_P^2) - wL}$$

Our A' corresponds to Jorgenson's variable A in (17) representing the presumed additional costs of adjusting the capital base; however, A' derives from the firm's adjusting its capital/output ratio for impacts of uncertainty in the factor/output price ratio. Note that A' has similar qualitative characteristics as A (see (13)) when I > 0, that is:

$$A' > 1\ , \ \frac{\partial A'}{\partial K} > 0,\ \frac{\partial A'}{\partial I} > 0$$

Then with only the same independence assumptions as Jorgenson, the OLSQ estimation of (30) would be indistinguishable from Jorgenson's (17) since (30) becomes:

$$\ln(\frac{F}{K}) = C' + \eta' \; [\ln(\frac{Z}{P}) + \ln A'] + \epsilon'_K \qquad (31)$$

where C' is a regression constant and η' is our regression coefficient.[21] Note that we have not needed to assume CES to derive (31) and our coefficient, η', is not to be interpreted as the elasticity of factor substitution. Jorgenson's specification of η in equation (17) as the elasticity of substitution depends critically on his additional maintained hypothesis of CES. Yet (31) gives the same estimation results as (17) and there is no independent test between the two specifications of CES. If we were to assume CES in our uncertainty model (31), then we have $C' = C$ and $\eta = \eta'$, thus the regression coefficient is correctly interpreted as the elasticity of substitution; however, it is dependent on the uncertainty aspects rather than costs of adjustment because A and A' are entirely different, though not mutually exclusive, concepts.

The empirical conclusions of Jorgenson, including his reconciliation[22] of all the other researchers' tests, hold equally well for our uncertainty model (31) for the output/capital ratio. This is the "identification problem" and the key question is obvious -- which model did he actually validate? Our specification from the CAPM, (31), is observationally equivalent to Jorgenson's model -- the same empirical tests would support either. In fact, with A' as a RHS variable, we would expect differing empirical estimates of η' in both cross-section and time series analyses, and for aggregate vs. micro models. For example, σ_{jM} is known to vary across time sub-periods and across industries. Thus, without the maintained hypothesis of CES, there would be no need for reconciliation of other researchers' results as proposed by Jorgenson. The literature contains no empirical exploration of the existence or order of magnitude of adjustment costs. We address this problem in a subsequent paper. As for the empirical relevancy of an individual firm's ability to affect cash flows of the whole market and σ_{jM} see Myers [31], Merton [28] and Rubinstein [34]. In U.S. capital markets such an ability is probably insignificant.

The identification problem just discussed is not dependent on the existence of non-competitive aspects of the capital market. We now show the same problem can exist in a perfectly competitive capital market in the CAPM but now with costs of adjustment. The value equation becomes:

$$V_j = \frac{1}{R}\left[\overline{P}F(K,L) - \overline{P}G(K,I) - wL + v(1-\delta)K - \lambda\sigma_{jM}\right] - I \tag{32}$$

The first order conditions are similar to (3) and (4) but with expected price:

$$\frac{\partial V}{\partial K} = 0 = \frac{\overline{P}F_K - \overline{P}G_K - \overline{P}G_I + v(1-\delta)}{R} - 1 \tag{33}$$

$$\frac{\partial V}{\partial L} = 0 = \frac{\overline{P}F_L - w}{R} \tag{34}$$

From (33) we have:

$$F_K = \frac{Z}{\overline{P}}\left[1 + \frac{\overline{P}G_I}{Z} + \frac{\overline{P}G_K}{Z}\right] \tag{33a}$$

Equation (33a) has the following interesting implication. In a world of uncertainty with a perfectly competitive capital market, the absolute amount of value-maximizing investment and output by the firm will be equal to that of a certain world. This is seen by comparing (33a) with (6). The firm does not take explicit account of risk. However, that firm's market value to risk averse investors will be: (a) strictly less (for $\sigma_{jM} > 0$) than the market value under certainty as seen by the required risk premium in equation (32); (b) strictly greater for $\sigma_{jM} < 0$; and (c) equal for $\sigma_{jM} = 0$.

In a manner similar to that of Section 2, we can use (33) and (34) to derive:

$$\ln\left(\frac{F}{K}\right) = -\ln\left[1 - \frac{wL}{\bar{P}F}\right] + \ln\left(\frac{Z}{\bar{P}}\right) + \ln\left[1 + \frac{\bar{P}G_I}{Z} + \frac{\bar{P}G_K}{Z}\right] \qquad (35)$$

This equation, (35), is identical to (12). Combining the first and last bracketed terms on the RHS of (35) yields the following OLSQ estimation equation:

$$\ln\left(\frac{F}{K}\right) = C'' + \eta''\left[\ln\left(\frac{Z}{P}\right) + \ln A''\right] + \varepsilon''_K \qquad (36)$$

where $A'' \equiv \left(\frac{\bar{P}F}{\bar{P}F - wL}\right)\left(1 + \frac{\bar{P}G_I}{Z} + \frac{\bar{P}G_K}{Z}\right)$. Again, with no assumptions regarding the type of production technology (except CRTS) and in a world of uncertainty with a competitive capital market, we have a general model which is empirically indistinguishable from the CES/certainty model, (17). Again there is no reason to restrict the interpretation of the regression coefficient in (36), η'', to that of an elasticity of substitution as required by the CES case.

Imposing CES on (36) has the same implications as we discussed following (31), where now $\eta'' = \eta$, the elasticity of substitution, but $A'' \neq A$.

Finally, Jorgenson discusses the results of Arrow et al. [1], Solow [38], Dhrymes and Zarembka [10], Bell [3], Fuchs [15], Griliches [17], and Lucas [26] where they all were estimating the elasticity of substitution from output/labor ratios. In this case Jorgenson says there are obviously no costs of adjustment since they are only relevant to capital. However, again the estimated elasticity of substitution differed significantly among researchers. In each case the model tested was the standard competitive one in a certain world:

$$F_L = \frac{W}{P} \tag{37}$$

Jorgenson explains the reported differences in results as due to bias resulting from the omission of international (and interregional domestic U.S.) differentials in labor quality and/or output price. He claims that adjustment for these differences, for example by adding a dummy variable to equation (37) as in Fuchs [15], will again reconcile all the differing estimates and the elasticity of substitution is, again, not significantly different from unity. But note from (25) in our uncertainty model we may write:

$$F_L = \frac{W}{\bar{P}}\left[\frac{\bar{P}}{\bar{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)}\right] \tag{38}$$

As we found for capital, F_L is greater[23] in uncertainty, (38), than for a certain world, (37), as we expect. This result implies that, with a noncompetitive capital market, labor employment and output will be strictly less under uncertainty than in a certain world due to the positive risk premium.

The output/labor equation derived from (37) for OLSQ estimation by the various researchers is:

$$\ln\left(\frac{F}{L}\right) = a_o + \eta \ln\left(\frac{w}{P}\right) + \varepsilon_L \tag{39}$$

where a_o is the constant term for the regression. To this specification Jorgenson suggests incorporation of another term, call it θ, on the right hand side to reflect the assumed international/interregional differences in labor quality and/or output price. The actual equation for estimation thus becomes:

$$\ln\left(\frac{F}{L}\right) = a_o + \eta[\ln\left(\frac{w}{P}\right) + \theta] + \varepsilon_L \tag{40}$$

Needless to say, θ may take on many different forms depending on the researcher since it is not specified by the theoretical derivation. However, recall our equation (38). In a manner similar to that we used in deriving (30) we can show the following:

$$\ln\left(\frac{F}{L}\right) = a'_o + \eta'''\left[\ln\left(\frac{w}{\overline{P}}\right) + \ln\left(\frac{\overline{P}}{\overline{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)}\right)\right] + \varepsilon'_L \tag{41}$$

The second term in square brackets in (41), $\ln\left(\frac{\overline{P}}{\overline{P} - \lambda(\sigma_{PM} + 2F\sigma_P^2)}\right) \equiv \theta'$, was derived entirely within our uncertainty model. It is qualitatively equivalent to the ad hoc θ used in (40). Note that our θ' varies with both $\overline{P}$ and L as θ is presumed to. The point is this. Equation (41) is as equally validated by Jorgenson's and the other researchers' empirical analyses as is (40). Again the identification problem--which model structure was actually tested?

4. INVESTMENT OPPORTUNITIES

We now extend the results of the previous uncertainty section in a fashion which explicitly illustrates the interaction considerations of a new investment opportunity, e.g., a new product. However, we use a similar methodology to ensure comparability of our findings with the previous analysis. All the assumptions made for our previous uncertainty model hold unless explicitly stated otherwise.

Consider a firm at the beginning of the period with no new investment opportunities. We may write:

$$\tilde{X}_j = [\tilde{P}F(K_o,L) - wL + v(1-\delta)K_o] \tag{42}$$

where:

$$E(\tilde{X}_j) \equiv \bar{X}_j = [\bar{P}F(K_o,L) - wL + v(1-\delta)K_o] \tag{43}$$

and L is the labor used by the existing (clay) technology capital, K_o, and as before:

$$\sigma_{jM} = F\sigma_{PM} + F^2\sigma_P^2 \tag{44}$$

Again, by the CAPM, we may substitute (43) and (44) into (19) to derive the equilibrium market value of the firm:

$$V_j = \frac{1}{R}[\bar{P}F(K_o,L) - wL + v(1-\delta)K_o - \lambda(F\sigma_{PM} + F^2\sigma_P^2)] \tag{45}$$

Now suppose this firm has a new investment opportunity to decide upon, with a different output technology represented by: $H \equiv H(I,N)$, where I represents the new (putty) capital goods' investment, N is the new labor required, and where:

$$H_I > 0, \quad H_{II} < 0, \quad H_N > 0, \quad H_{NN} < 0.$$

Assuming, for expositional simplicity, v, w, δ and $\tilde{P}$ to be equal for both types of output technologies, we have:

$$\tilde{X}'_j = [\tilde{P}F(K_o,L) + \tilde{P}H(I,N) - w(L+N) + v(1-\delta)(K_o+I)] \tag{46}$$

and therefore:

$$E(\tilde{X}'_j) \equiv \bar{X}'_j = [\bar{P}F(K_o,L) + \bar{P}H(I,N) - w(L+N) + v(1-\delta)(K_o+I)] \tag{47}$$

In similar fashion to (22), the covariance term after taking the new investment becomes:

$$\sigma'_{jM} = \text{cov}[\tilde{P}F(K_o,L) + \tilde{P}H(I,N) - w(L+N) + v(1-\delta)(K_o+I),$$

$$\tilde{X}_M + \tilde{P}F(K_o,L) + \tilde{P}H(I,N) - w(L+N) + v(1-\delta)(K_o+I)]$$

$$= F\sigma_{PM} + F^2\sigma_P^2 + 2FH\sigma_P^2 + H\sigma_{PM} + H^2\sigma_P^2 = \sigma_{jM} + \sigma_{HM} + \sigma_{Hj} + \sigma_H^2 \tag{48}$$

where, as before, $\tilde{X}_M$ represents the market cash flow before the new investment is undertaken. As discussed previously, we assume the firm perceives aggregate investment in the new technology to be affected by its decision.

Substituting (47) and (48) into (19), the market value of firm j including the new project is:

$$V'_j = \frac{1}{R}[\bar{P}F(K_o,L) + \bar{P}H(I,N) - w(L+N) + v(1-\delta)(K_o+I) -$$

$$- \lambda(F\sigma_{PM} + F^2\sigma_P^2 + 2FH\sigma_P^2 + H\sigma_{PM} + H^2\sigma_P^2)] \tag{49}$$

It is fruitful to break out the specific effects and implications of the investment decision. The firm's objective is to choose I such that the change in market value less the requisite investment, $\Delta V_j = V_j' - V_j - I$, is maximized (subject to the constraint $I \geq 0$). The firm is assumed to have already maximized (45) with respect to its choices of K_o and L. Subtracting (45) and I from (49) we get:

$$\Delta V_j = \frac{1}{R}\,[\overline{P}H(I,N) - wN + Iv(1-\delta) - \lambda(2FH\sigma_P^2 + H\sigma_{PM} + H^2\sigma_P^2)] - I \qquad (50)$$

The necessary conditions for a maximum of (50) are:

$$\frac{\partial \Delta V_j}{\partial N} = 0 = \frac{1}{R}\,[\overline{P}H_N - w - \lambda(2FH_N\sigma_P^2 + H_N\sigma_{PM} + 2HH_N\sigma_P^2)] \qquad (51)$$

$$\frac{\partial \Delta V_j}{\partial I} = 0 = \frac{1}{R}\,[\overline{P}H_I + v(1-\delta) - \lambda(2FH_I\sigma_P^2 + H_I\sigma_{PM} + 2HH_I\sigma_P^2)] - 1 \qquad (52)$$

Solving (52) for H_I, the marginal product of investment in the new technology, and substituting from (48) in the covariance term gives:

$$H_I = \frac{R - v(1-\delta)}{\overline{P} - \frac{\lambda}{H}(\sigma_{HM} + \sigma_{Hj} + 2\sigma_H^2)} \qquad (53)$$

Compare (6), (27), and (33a) with (53). We see that in all three models, the firm chooses new investment such that its marginal product is qualitatively very similar. In particular, the following characteristics of H_I in (53) are identical to those of Jorgenson's model in (8):

$$\frac{\partial H_I}{\partial r} > 0,\ \frac{\partial H_I}{\partial \overline{P}} < 0,\ \frac{\partial H_I}{\partial v} < 0,\ \frac{\partial H_I}{\partial \delta} > 0.$$

It is clear that Jorgenson's empirical analysis of (6) which we found validates (27) and (33a) will be equally valid for (53).

From Jorgenson's model, (6), the larger is G_I, the larger will be F_K and therefore the smaller will be new investment. The more closely (causally) related are the firm's existing assets with the new project, the larger will be G_I. This implies the firm takes less of the new investment. Our model yields the same result. In (53) the σ_{Hj} term enters positively[24] which implies $\frac{\partial H_I}{\partial \sigma_{Hj}} > 0$. Thus the more closely (positively) correlated are existing assets' cash flows with those of the new investment, the less the firm will invest in it.[25] This seeming "diversification" motive is actually due to our assumption that the firm perceives itself as affecting aggregate project investment and thus the cost of capital, as discussed previously. This latter effect is the correct explanation of the Jensen and Long "surprising" result regarding differential investment demand across firms due to "something akin to a diversification consideration" (See [18], pp. 158-9).

Recall that the source of uncertainty in our model is consumer tastes and demand as represented by $\tilde{P}$ in the output market. Thus, in a more general case, suppose the new investment's output is in some way differentiated from the existing assets' output and is therefore priced by consumers in a different market represented by $\tilde{P}^H$, versus $\tilde{P}^F$ in our earlier case of homogeneous output. Then σ_{Hj} depends solely on the relationship between $\tilde{P}^H$ and $\tilde{P}^F$, that is on how closely related the output markets are vis-a-vis consumer demand. The point is that, given the assumptions of our model, although the firm theoretically invests less in a project with larger σ_{Hj}

(i.e., more closely correlated with existing assets), this does not necessarily imply the firm will pursue conglomerate diversification, in terms of project relative size, at the output product level. Rather it will simply invest more in products for consumer markets with less-related output pricing structures. For example, it would be perfectly consistent for a firm to invest in both industrial adhesives and surgical adhesive tape. We would not consider this conglomerate diversification. But at first glance this empirical fact might seem to conflict with our result in (53) due to the existence of σ_{Hj}. But of course industrial adhesives and surgical tape are priced in totally different consumer markets and thus the correlation[26] between $\tilde{P}^H$ and $\tilde{P}^F$ may be quite low, implying low H_I as a result of higher investment. These results are partially supported by the empirical work of Berry [4]. He found little new intrafirm investment occurring in related (to existing assets) 4-digit SIC index industries, but large amounts within the same 2-digit classification. Of course this same empirical fact lends support to the costs of adjustment model if we believe related 4-digit SIC industries have larger G_I in equations (6) and (33a).

We can also make comparative statements about output in these models. In a world of certainty with no costs of adjustment, (6) and (53) both become:

$$F_K = H_I = \frac{Z}{P} \qquad (54)$$

By inspection, H_I in (54) is smaller than in either (6) or (53). Given our production function assumptions (and that $\sigma_{Hj} \geq 0$ in general), this implies that both investment and output are larger in (54) -- that is, output under uncertainty for this firm is strictly less than output under certainty. Also, as the distribution of $\tilde{P}$ becomes "tighter" (i.e., as $\sigma_P^2 \rightarrow 0$) the $(\sigma_{HM} + \sigma_{Hj} + 2\sigma_H^2)$ term in the denominator of (53) goes to zero also and output and investment approach that of certainty. This does not necessarily imply that a monopolist, who can exercise some control over the distribution of output price, will invest and produce more than the competitive firm in an uncertain world.

APPENDIX

Jorgenson assumes constant returns to scale (see his equation (6) on page 224 of [19]), i.e., the production function is linearly homogeneous.

If we assume a competitive environment so that factors are paid their marginal product, then Euler's theorem holds and the total product will be exhausted exactly by the distributive shares for the inputs, capital and labor. Thus, applying Euler's theorem, using Jorgenson's otation, we have:

$$K \frac{\partial Q}{\partial K} + L \frac{\partial Q}{\partial L} = Q \qquad \text{(A1)}$$

where $K \frac{\partial Q}{\partial K}$ is the share of capital and $L \frac{\partial Q}{\partial L}$ is labor's share. Substituting into (A1) from Jorgenson's equation (7), in [19] p. 225, we have:

$$\frac{K}{Q}\left[\delta\gamma^{-\rho}\left(\frac{Q}{K}\right)^{1+\rho}\right] + \frac{L}{Q}\left[\gamma^{-\rho}(1-\delta)\left(\frac{Q}{L}\right)^{1+\rho}\right] = 1 \qquad \text{(A2)}$$

Rearranging and combining terms in (A2) gives:

$$1 - \frac{L}{Q}\left[\gamma^{-\rho}(1-\delta)\left(\frac{Q}{L}\right)^{1+\rho}\right] = \delta\gamma^{-\rho}\left(\frac{Q}{K}\right)^{\rho} \qquad \text{(A3)}$$

But from Jorgenson's equation (7) we know:

$$\left[\gamma^{-\rho}(1-\delta)\left(\frac{Q}{L}\right)^{1+\rho}\right] = \frac{q_L}{q_Q} \qquad \text{(A4)}$$

Substituting from (A4) into (A3) gives:

$$1 - \frac{q_L}{q_Q}\frac{L}{Q} = \delta\gamma^{-\rho}(\frac{Q}{K})^{\rho} \tag{A5}$$

or in our notation:

$$1 - \frac{wL}{PF} = \delta\gamma^{-\rho}(\frac{Q}{K})^{\rho} \tag{A5'}$$

where the right hand side term in (A5') is the relative share of capital.* That this relative share equals $\frac{ZK}{PF}$ in equation (15) can be shown from Jorgenson's equation (7) as:

$$\delta\gamma^{-\rho}(\frac{Q}{K})^{\rho} = \frac{K}{Q}\;\frac{q_K}{q_Q} \;=\; \frac{ZK}{PF} \tag{A6}$$

where the last term in (A6) is simply my notation. Substitution of (A6) into (A5') shows:

$$1 - \frac{wL}{PF} = \frac{ZK}{PF} \tag{A7}$$

It can be shown** that if this relative share of capital is constant, the elasticity of substitution must be equal unity.

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* See (A2) above where $\frac{K}{Q}\;\frac{\partial Q}{\partial K} = \frac{K}{Q}\left[\delta\gamma^{-\rho}(\frac{Q}{K})^{1+\rho}\right] = \delta\gamma^{-\rho}(\frac{Q}{K})^{\rho}$.

** See, e.g., Allen, R. G. D., Mathematical Analysis for Economists, pp. 363-4.

FOOTNOTES

1. See Eisner and Strotz [12], Lucas [25], Uzawa [41], Treadway [40], Gould [16], and Rothschild [33].

2. See Rothschild [33], and Jorgenson [19].

3. As derived by Sharpe [36 and 37], Lintner [23 and 24], and Mossin [30], and extended by Black [7], Fama [13], and Merton [27].

4. Value additivity with respect to new investment projects holds based on the assumption (among others) that the new project's cash flow distribution is causally independent of the firm's existing and potential other projects. Causal independence is thus a necessary condition for value additivity.

5. Rothschild's model assumes a monopolist with a strictly concave (in output) revenue function. However, his adjustment cost proofs follow as well for the competitive case.

6. Jorgenson's assumption of a putty-clay model seems to be quite realistic empirically though theoretically "short-run."

7. We assume no taxes and although our model may also represent I as borrowed funds rather than equity, there is no real distinction among financial claims in our certain world.

8. We assume v is net of the rental rate on capital which is an implicit user cost on capital owned by the firm. This rental rate is an opportunity cost -- the firm could rent out all K_o rather than use it in production.

9. These results are comparable to the dynamic Euler conditions which Jorgenson [19] adapts from Lucas' [25] use of the calculus of variations. These conditions are satisfied at each point in time over the optimal program.

10. Arrow et. al. [1], Dhrymes [9], Dhrymes and Zarembka [10], Fuchs [15], Bell [2], Griliches [17], Kmenta [21], and Bischoff [5].

11. The elasticity of substitution among factors is defined as the percentage change in the factor input ratio with respect to a percentage change in their marginal rate of substitution (= their factor cost ratio in perfect competition).

12. Actually $A \gtreqless 1$ as $G_I \gtreqless G_K$, but the meaningful presumption on costs of adjustment is that they are positive, and we generally expect $G_I >> G_K$ and therefore $A > 1$. Also note that equation (12) is not the reduced form, even though Jorgenson transforms it into equation (17) for the empirical estimation. This is because A contains G_I and G_K which are functions, at least, of K, a left hand side variable.

13. [19], p. 224.

14. [19], p. 234.

15. Recall Jorgenson's maintained hypothesis of CRTS.

16. The CAPM is a static (single-period) model but can be shown to hold intertemporally under certain assumptions as discussed in Fama [13] and Merton [27]. Basically we require the investment opportunity set and investors' attitudes toward risk be constant over time. In our current analysis this requirement translates into $R_t \overset{t}{\equiv} R$ and $\lambda_t \overset{t}{\equiv} \lambda$, i.e., the riskless rate and market price of risk are constant over time.

17. Equation (19) is derived as follows. By the CAPM:

$$\bar{R}_j = R + \frac{\sigma_{jM}}{\sigma_M^2}[\bar{R}_M - R] \Longrightarrow$$

$$\Longrightarrow \frac{\bar{X}_j}{V_j} = R + \frac{\text{cov}\left(\frac{\tilde{X}_j}{V_j}, \frac{\tilde{X}_M}{V_M}\right)}{\text{var}\left(\frac{\tilde{X}_M}{V_M}\right)}\left[\frac{\bar{X}_M}{V_M} - R\right] = R + \frac{\frac{\text{cov}(\tilde{X}_j,\tilde{X}_M)}{V_jV_M}}{\frac{\text{var}(\tilde{X}_M)}{V_M^2}}\left[\frac{\bar{X}_M - RV_M}{V_M}\right] \Longrightarrow$$

$$\Longrightarrow \frac{\bar{X}_j - RV_j}{V_j} = \left(\frac{\sigma_{jM}}{V_j}\right)\left(\frac{V_M}{\sigma_M^2}\right)\left(\frac{\bar{X}_M - RV_M}{V_M}\right) = \frac{\sigma_{jM}}{V_j}\underbrace{\left(\frac{\bar{X}_M - RV_M}{\sigma_M^2}\right)}_{\equiv \lambda} = \frac{\lambda\sigma_{jM}}{V_j} \Longrightarrow$$

$$\Longrightarrow \bar{X}_j - RV_j = \lambda\sigma_{jM} \Longrightarrow V_j = \frac{1}{R}[\bar{X}_j - \lambda\sigma_{jM}].$$

18. For a more detailed discussion of this methodology see Jensen and Long [18] or Merton and Subrahmanyam [29].

19. Since the CAPM assumes investors are risk-averse, we have $\lambda(\sigma_{PM} + 2F\sigma_P^2) > 0$ for a non-Giffen good, i.e., $\sigma_{PM} \geq 0$.

20. For a complete analysis and discussion see Merton and Subrahmanyam [29]. Also see footnote 25.

21. Equation (31) is not the reduced form since A' is a function of F, a left hand side variable. This also will be the case for A" in equation (36) and θ' in equation (41). However, we leave them in this form so as to have them comparable with Jorgenson's model, equation (12), which has a similar relationship in K -- see footnote (12).

22. For example, in [19], p. 234, his analysis of misspecification bias from ln A holds in (31) as does the error term correlation with ln (F/K) in a "reversed" version of (31).

23. Note the bracketed term in (38) is greater than unity.

24. We assume that, in general, investment projects are positively correlated.

25. Empirically, the magnitude of σ_{Hj} and σ_H^2 in (53) are much less than σ_{HM} as shown by Merton [28]. That is, σ_H^2 is on order of the size of the project and σ_{Hj} is on order of the size of the firm, but σ_{HM} is on order of the size of the whole market. Thus, although our results are completely valid in theory, the practical significance that the firm actually takes σ_{Hj} and σ_H^2 into account in investment decisions may be quite small.

26. Note that for simplicity we have assumed only demand uncertainty. In a more complete model, variables other than price may be random, and price itself may depend on sources of uncertainty other than demand -- random cost elements for example -- which may be highly correlated across products.

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*Key to source abbreviations:

AER: American Economic Review.
Bell Journal: The Bell Journal of Economics.
IER: International Economic Review.
JOB: Journal of Business.
JOF: Journal of Finance.
JPE: Journal of Political Economy.
QJE: Quarterly Journal of Economics.
RES: Review of Economic Studi
RESTAT: Review of Economics and Statistics.
SEJ: Southern Economic Journa

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